

MILLIMAN RESEARCH REPORT

Fixed indexed annuities with Market Risk Benefits

Cash flow modeling: Can we keep it simple?

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Executive Summary

The release of Financial Accounting Standards Board (FASB) Accounting Standards Update 2018-12, Targeted Improvements for Long-Duration Contracts (ASU 2018-12, also referred to as LDTI) has created a flurry of activity in the life insurance industry, as actuaries and other professionals try to interpret and apply the new accounting standard.

Among other major changes, the standard creates a new category of liabilities for “Market Risk Benefits” (MRBs), which must be held at fair value, as defined under existing GAAP standards. The definition of an MRB encompasses, among other possibilities, all kinds of guaranteed living benefits and guaranteed death benefits (GMxBs) on deferred annuity contracts, including both variable annuities (VAs) and fixed indexed annuities (FIAs). Under current, pre-LDTI GAAP, many kinds of GMxBs are valued under an insurance benefit model, with reserves calculated under a Statement of Position (SOP) 03-1 methodology and typically not using market-consistent assumptions. Thus, the new fair value MRB model represents a significant change for some products, with both financial and operational implications.

Based on our discussions with clients, the valuation of MRBs on FIA contracts is an area where we see a wide range of approaches emerging and a lack of industry consensus at this time. This paper, the first in a series, aims to provide insights that will be helpful to our clients as they shape their new valuation methodologies for MRBs on FIA contracts. This paper focuses on the cash flow modeling aspects of MRBs on FIA contracts, especially the methodology for projecting indexed account growth.

We discuss several emerging approaches to modeling option budgets for MRB calculations, including some arguments for or against each. We describe two methods for projecting index credits, the option budget crediting method and the stochastic equity crediting method. We discuss some challenges in designing a market-consistent stochastic framework for FIAs with GMxBs, highlighting how it can be a very complex exercise. We conclude with an analysis of an FIA with a guaranteed lifetime withdrawal benefit (GLWB) rider, comparing the results of a deterministic approach using the option budget crediting method to several stochastic approaches, as a way of evaluating whether any of the stochastic approaches are worth the effort.

Our analysis showed that the deterministic methodology produces results that are nearly identical to the results of any of the stochastic methodologies, both in terms of baseline results and sensitivity to market factors. The results indicate significant market sensitivity, but very little equity or interest rate optionality. This sort of analysis, tailored to a company’s specific products and assumptions, could provide justification to use the deterministic approach to valuing MRBs on FIAs. The simpler approach will reduce complexity and computation cost and time, and in many cases will make it easier to achieve consistency between existing FAS133-type liabilities and MRB liabilities.

Introduction

Liabilities for MRBs must be held at fair value under the new standard. Under U.S. GAAP, fair value is defined as an estimate of the exit price for an asset or liability on the valuation date, assuming an orderly transaction between market participants in the principal market for the asset or liability. When fair value is not directly observable and must be estimated, the guidance emphasizes the use of market assumptions over entity-specific assumptions, and the use of observable inputs over unobservable inputs.

The fair value guidance suggests three possible valuation techniques, which it calls the market approach, the cost approach, and the income approach. As far as we are aware, the income approach is by far the most common approach employed for valuing insurance liabilities held at fair value under current GAAP, and we expect it to be the standard approach for valuing MRBs under LDTI. Under the income approach, fair value is estimated as a risk-adjusted expected present value of future cash flows from the asset or liability. The guidance for the income approach suggests several components of the risk-adjusted expected present value under the income approach:

- Expectations about future cash flows and possible variation in the timing and amount of those cash flows.
- Time value of money, using an appropriate discount curve.
- The price for bearing the uncertainty in the cash flows (i.e., a risk premium or risk margin).
- For a liability, the nonperformance risk of the obligor (e.g., own credit risk).

Each of these components is discussed in the guidance, but the guidance is non-prescriptive and includes no specifics for insurance contracts.

Applying fair value methods to value guaranteed minimum benefits is not new to the annuity world. Guaranteed minimum accumulation benefits (GMABs) and guaranteed minimum withdrawal benefits (GMWBs) on variable annuities are usually held at fair value under current GAAP. Insurers also employ market-consistent methods that are similar to fair value when hedging guaranteed minimum death and income benefits (GMDBs and GMIBs) as well as GMABs and GMWBs. These VA GMxB calculations typically use risk-neutral methods based on option pricing theory.

However, GMxBs on FIAs are quite distinct from GMxBs on VAs for the reasons detailed below.

- Returns on indexed strategies are floored at zero and often capped for a single index segment term, unlike VAs, which can experience unlimited account value loss and growth. FIAs therefore have a narrower return distribution over the short and long term.
- On most FIA products the insurer has the ability to reset the index cap, spread, and participation rates, and the option budgets used to determine the reset rates are a primary driver of indexed account performance. The insurer therefore exerts influence over account value growth for the FIA in a way that it does not for VA separate account returns.
- Unlike equity and bond returns, which are driven by the same factors for all investors, the level of option budgets and even the approach to determining option budgets varies from company to company and is thus highly subjective.
- Many companies set option budgets with consideration of general account book yields, which are not usually modeled explicitly in risk-neutral frameworks for VA business.
- The pricing of embedded options for index crediting features adds another level of complexity for stochastic FIA modeling.

Hence, there are several additional considerations and complications an actuary or insurance entity would need to consider when valuing MRBs in FIAs. Of particular concern is the approach to modeling indexed account growth, which may be thought of as two major choices: (1) how to project the option budget, and (2) how to project index credits given the option budget. These two choices are discussed in the following two sections.

Modeling the option budget

On many FIA contracts, the cap rates, participation rates, spreads, and other parameters for determining index credits are a non-guaranteed element that the insurer has the right to reset periodically, usually subject to guaranteed minima or maxima. Insurers typically reset the index strategy rates such that the market cost of replicating options is equal to a specified budget. Because the indexed returns are a primary determinant of the future GMxB claims, the option budget is a key assumption for valuing the MRBs on an FIA contract. This ability to reset the index strategy rates gives the insurer control over the expected indexed account returns. It also introduces an extra wrinkle that is not present in most option pricing problems where the parameters are fixed or at least mechanically defined at inception, and not unilaterally adjustable by one party.

Insurers have different philosophies for determining option budgets. The most common approach is to deduct a profit spread from the book yield on the general account assets supporting the business. This approach is straightforward to model, requiring only a projection of book yields and an assumption for the profit spread. The projected book yields can be developed from asset projections or using simplified approaches that start with the current book yield and consider expected reinvestment rates. The profit spreads are readily available from pricing or deducible from actual book yields and hedge spend in recent periods.

How, if at all, should the modeling approach change in the context of a GAAP fair value calculation? We see several competing approaches emerging as companies begin to design their FIA MRB frameworks, which we describe below. These approaches vary in how they incorporate market rates, and to what extent they recognize typical industry practice or actual practice at the issuing company.

- The first approach suggests that the option budget should be directly tied to (possibly stochastic) market-implied forward interest rates (i.e., an immediate linear relationship between interest rates and option budgets—if risk-free rates fall by 50 basis points, then the modeled option budgets would also decrease by 50 basis points, all else equal). Different formulations are possible within this family, with adjustments to the risk-free rate that may include a positive spread for illiquidity or risk-adjusted credit spread, and/or a negative spread for insurer profits. This approach is consistent with the idea of resetting option budgets to be consistent with risk-adjusted new money yields available to market participants. Some consider the direct use of market rates to be an essential requirement for market consistency, but for most companies this will be a departure from their actual practices and from industry practice, which usually emphasizes in-force book yields rather than new money yields.
- A second approach suggests that option budgets can still be modeled using the book-yield-less-spread approach, but using market-implied forward interest rates and market spreads as inputs into the projection of future book yields. For example, a company may start with its current book yield and project that yield forward assuming reinvestment based on the forward risk-free rate curve and risk-adjusted credit spreads. The same approach can be applied in a stochastic interest rate model, replacing the deterministic forward curve with stochastic interest rates calibrated to risk-free rates and interest rate option prices. This approach will be closer to actual practice for many companies that set budgets in reference to book yields, but it is more dependent on entity-specific assumptions (existing asset yields, reinvestment mix) than the first approach.
- A third possible approach is to use fixed or possibly time-varying budgets that reflect management's expectation as of the valuation date. For some companies this will be similar to their actual rate-setting practices, where planned budgets are determined at issue and updated infrequently. It may also give a result similar to the second approach, because book yields on single premium contracts will usually be very stable, particularly when asset and liability cash flows are well matched. This approach is, however, even more dependent on entity-specific assumptions than the second approach.
- A fourth approach is to grade between current option budget (i.e., the actual budgets the company uses to set caps as of the valuation date) and a budget derived as in the first approach (directly linked to forward interest rates). This has been proposed by some as a compromise between reflecting actual practice on the one hand and maximizing the use of market rates on the other hand.

It is important to highlight that formulations that are directly tied to market rates, as in the first approach, will create additional interest rate sensitivity in the results, and the results will already be highly interest-rate-sensitive due to discounting effects. Some would argue that the additional interest rate sensitivity due to changing option budgets under the first type of approach is somewhat artificial given that actual option budgets tend to be relatively stable, especially over short periods of time.

Another important consideration in the development of the option budget projection methodology is consistency with the assumptions used to value the embedded derivative for the index-crediting feature on the base contracts (often referred to as the "FAS133" embedded derivative). The new standard does not modify the requirements of that calculation, which is also a fair value calculation that requires a projected option budget. While there could be some arguments for different budgets (e.g., lower budgets as a risk margin in the MRB calculation), we expect that most companies will strive to achieve a consistent approach for determining budgets for the MRB and the index-crediting embedded derivative.

Projecting indexed account growth

Once the option budgets have been derived, there are two diverging approaches to projecting index credits, which we will refer to as the “stochastic equity crediting method” and the “option budget crediting method.”

OPTION BUDGET CREDITING METHOD

Under the option budget crediting method, index credits are based directly on the option budgets. In a market-consistent, risk-neutral version of this method, open segments are marked to market as of the valuation date and accumulated at risk-free rates between the valuation date and expiry. Subsequent index credits are modeled as the option budget accumulated at risk-free rates between the start and end of an index segment.

The rationale for this approach is that, in a risk-neutral valuation setting, the discounted expected payoff of a strategy must be exactly equal to the cost (or value on the valuation date for open segments), otherwise the framework is not arbitrage-free. The option budget method enforces this condition directly. The shortcoming of this approach is that it does not quantify the full distribution of results. To the extent there is any asymmetry in the actual distribution of payoffs, that asymmetry will not be measured.

We note that the option budget crediting method is usually used with a single deterministic scenario, but it can also be used with stochastic interest rate scenarios where the budgets and other interest-sensitive factors (e.g., dynamic lapse) vary by scenario.

STOCHASTIC EQUITY CREDITING METHOD

Under the stochastic equity crediting method, index credits are projected using stochastic equity returns. Cap, participation, and spread rates are determined at reset dates based on the projected option budget and projected market parameters.

This method requires a market model with stochastic equity returns, and an algorithm for calculating the cap, participation, and spread rates at reset dates. In a risk-neutral, arbitrage-free framework, there must be a consistency between the market model and the option pricing algorithm, in order to ensure that the discounted expected payoff on forward starting index strategies equals the option budget.

The stochastic crediting method comes with significant additional complexity over the option budget crediting method, and also over the typical market-consistent, risk-neutral methodology employed to value VA GMxBs. We describe some of these issues below.

- For market-consistent valuation of GMxBs on VA business, it is common to simplify the volatility structure reflected in the equity scenarios. For example, many companies calibrate their risk-neutral scenarios for VA GMxBs using at-the-money volatility that varies with time only (i.e., volatility with term structure but without skew). However, this simplified model will not be able to reproduce market prices for open segments on even vanilla strategies, due to lack of volatility skew. This is especially true for cap strategies, which constitute a large portion of the exposure at many companies. For non-vanilla strategies such as cliquets and other path-dependent options, stochastic volatility models are required to replicate market prices.
- As noted above, the function for pricing forward starting options is dependent on the market model. It is only appropriate to use the Black-Scholes formula to price the forward starting options when the market model is consistent with the assumptions underlying Black-Scholes, or when it can be demonstrated that the Black-Scholes price is a close approximation to the no-arbitrage price. Models with nonconstant but deterministic volatility require integration of the instantaneous forward volatility to convert it to a Black-Scholes implied volatility, and stochastic volatility models usually require numerical methods to calculate option prices. Such methods are usually not available off the shelf in actuarial projection software.
- Some companies plan to include stochastic interest rates in their frameworks, because they believe their FIA MRBs contain interest rate optionality from crediting guarantees or policyholder behavior. Stochastic interest rates further complicate the option pricing math, as the interest rate variance contributes to equity variance under risk-neutral scenarios.
- Most actuarial software uses monthly (or less frequent) time steps for liability projections and does not easily handle daily variation in strike price or exact expiration dates that do not align with calendar month-ends.

- Many FIA writers use a mix of common public indices and bespoke indices developed by or with investment banks specifically for FIA products. It is common for a company to have more than 10 indices offered across its entire FIA book. This creates the need to model correlations among indices. This can be difficult either on a market-consistent basis (due to paucity of instruments that reveal market-implied correlations) or on a historical basis (due to relatively short histories for bespoke indices; back-casting may or may not be feasible).

Some of these challenges can be resolved by using different approaches for modeling open segments and future starting segments. For example, open segments may be valued outside of the actuarial software, using more complex models and volatility parameters as may be necessary to fit market prices well. Then future starting segments may use simpler models and volatility structures that capture only the market features that drive the distributions of returns, insurer actions, and policyholder behavior. Some companies may choose to further simplify the calculations for future starting segments by mapping non-vanilla strategies to vanilla strategies or low-volume indices to higher-volume indices.

Even with simplifications, we believe there is a lot of room to design or implement a framework that does not maintain the critical expected-payoff-equals-budget condition, introducing unintended bias or noise to the MRB results. This raises the question, why not skip all of the complexity and simply enforce that condition directly through the option budget crediting method? The answer to that question, we believe, lies in whether a particular product really has optionality that requires stochastic methods to quantify. In the next section, we explore that question for an illustrative product and assumption set.

Comparison of deterministic and stochastic models

As highlighted above, a full-blown stochastic model that is market-consistent and arbitrage-free can be quite complex. To help evaluate whether the stochastic approach is really worth this additional effort, we modeled a representative product under four different market models, ranging from simple (the deterministic option budget method) to relatively complex (stochastic equity and stochastic interest rates).

MARKET MODELS

The table in Figure 1 summarizes the four market models that we included in our analysis.

FIGURE 1: FOUR MARKET MODELS USED IN ANALYSIS

Model	Equity Returns	Interest Rates	Index Crediting
Deterministic	N/A	Deterministic	Option budget method
Stochastic Rates	N/A	Hull White	Option budget method
Stochastic Equity	Lognormal w/ constant vol	Deterministic	Stochastic equity method
Stochastic Rates and Equity	Lognormal w/ constant vol	Hull White	Stochastic equity method

Each model was “calibrated” to the same hypothetical market environment with:

- Constant 3% forward interest rates
- Constant 15% equity implied volatility (applicable only for the two stochastic equity models)

In the first two models, where we did not use stochastic equity scenarios, the index credits are modeled as the option budget accumulated at risk-free rates between the start of an index segment and its expiry.

For the models with stochastic equity returns, forward starting cap rates were calculated using the Black-Scholes formula,¹ based on the projected option budget, interest rates, and implied volatility at the reset dates. We applied the cap rates to the equity index growth rates to calculate the interest credited.²

The market models and their illustrative parameters are specified in the appendix. We chose to illustrate relatively simple dynamics for equity returns and interest rates (e.g., constant volatility and constant expected forward rates), primarily to keep the option pricing math tractable. While these illustrative models are not capable of producing a close fit to a rich set of market features that may be relevant to FIA MRB valuation (e.g., equity volatility term structure and skew, swaptions implied volatility cube, etc.), we believe the particulars of the equity and interest rate distributions are not important to answer the more basic question: is it necessary to use a distribution at all?

ILLUSTRATED PRODUCT FEATURES

The illustrated product is an FIA contract with one-year, annual reset cap strategy, and a GLWB rider. The illustrated GLWB rider has an annual benefit base compound roll-up of 7% until activation of the benefit and an annual rider charge of 1%. The maximum withdrawal percentage is a function of the policyholder’s attained age at activation.

ILLUSTRATED ASSUMPTIONS

The actuarial assumptions used in this analysis are described in the appendix. We highlight two key assumptions that have an effect on the market sensitivity of the value of the GLWB rider.

- Option budgets are a function of projected interest rates, using a set of formulae that are intended to approximate a book-yield-less-spread approach, assuming a starting portfolio of fixed yield and a percentage of the portfolio turns over each period at projected new money rates. Thus, option budgets vary in the stochastic interest rate models and in the interest rate sensitivities, but do not immediately reset to new

¹ We assumed the Black-Scholes implied volatility is equal to the volatility parameter of the equity model. For the relatively simple dynamics and parametrizations of the illustrated market models, the resulting Black-Scholes price is either the exact arbitrage-free price or an extremely close approximation (price error less than 1 basis point).

² The dividend yield was assumed to be zero in the projected equity returns used to simulate the index payoffs and also in the Black-Scholes formula used to price the caps.

money rates as in the first approach described in the earlier section on option budgets. The modeled approach is of the second variety described in that section.

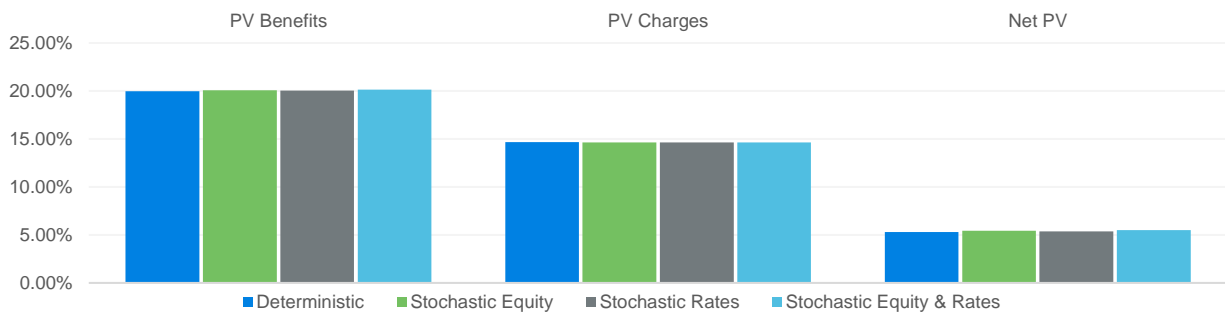
- Projected surrender rates are calculated as base lapse rates that are dynamically adjusted by comparing the present value of expected benefits to the surrender value. The present values of expected benefits are calculated using projected interest rates, so this dynamic lapse formula is a function of interest rates as well as equity performance.

RESULTS

Baseline results

Figure 2 shows the unitized expected present values for each market model, under the baseline market parameters. The present values are calculated as of the issue date.

FIGURE 2: BASELINE RESULTS AS % OF INITIAL PREMIUM



Under the baseline market parameters, the results are nearly identical across all four market models. This suggests that, for this product design and assumption set, there is little optionality with respect to interest rates and equity performance embedded in the GLWB rider. Further, this result indicates that the deterministic approach is a good estimator of any of the stochastic approaches when they are calibrated to the same market environment.

As a way of explaining this result, consider Figure 3. The green line shows the expected GLWB claims as a function of the indexed account growth rate. The blue histogram shows the probability density of the average indexed account growth rates, measured over 20 years, in the stochastic rate, stochastic equity model.

FIGURE 3: EXPECTED GLWB CLAIMS VS. AVERAGE INDEXED ACCOUNT GROWTH

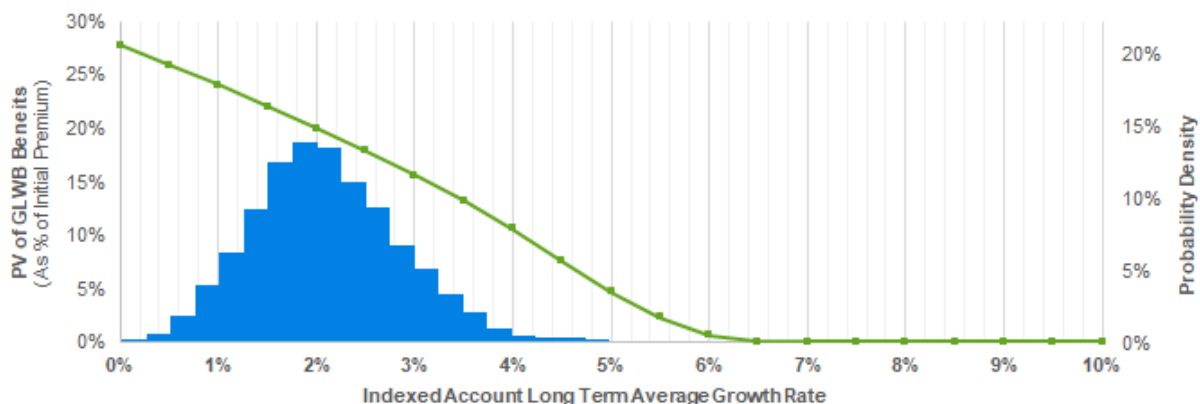


Figure 3 reveals several key features for the illustrated product:

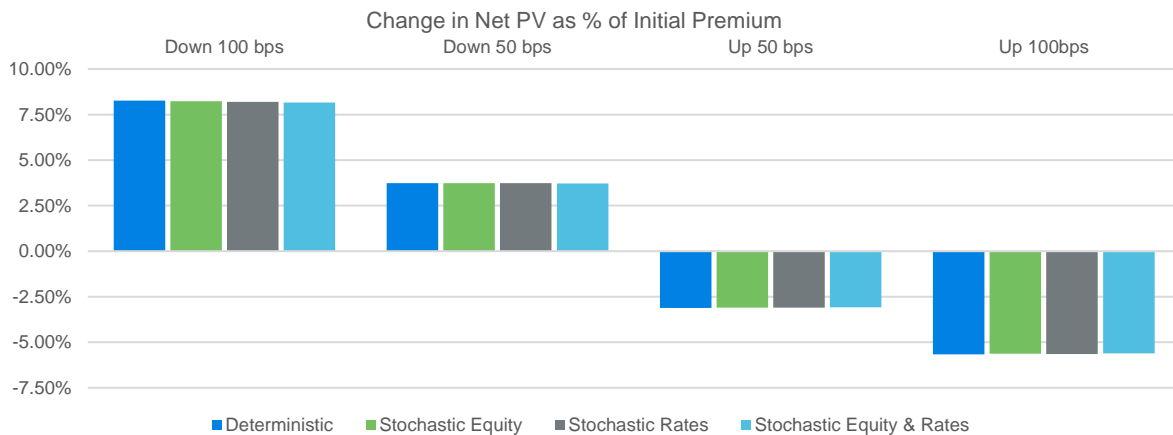
- The distribution of indexed account returns over a 20-year period is relatively narrow. For comparison, distribution of average returns for a VA invested in 70% equities and 30% cash would range from -7% to +10%, for the same market model and parameters.
- The distribution of index payoffs is approximately normal, nearly symmetric, and centered around the deterministic option budget of 2%.
- The GLWB payoff is almost perfectly linear with respect to the indexed growth rate over the entire range of possible outcomes in the model.
- At issue, the GLWB is deep in-the-money. In fact, the GLWB does not go completely out-of-the-money for all cells in any of the modeled scenarios.

While technically the GLWB is a put on the indexed growth rate over the life of the contract, the convexity in the payoff (i.e., the kink in the green line near 6% growth rates) is practically irrelevant due to the limited upside. This explains the apparent lack of option value exhibited in the results shown in Figure 2.

Interest rate sensitivity

Figure 4 shows the change in unitized net present values, compared to the baseline results, for several shocks to the level of interest rates.

FIGURE 4: INTEREST RATE SENSITIVITY



The GLWB claims are very long dated (weighted average life = 28 years), and hence there is naturally a significant amount of rate sensitivity from pure discounting effects. Additional sources of rate sensitivity are:

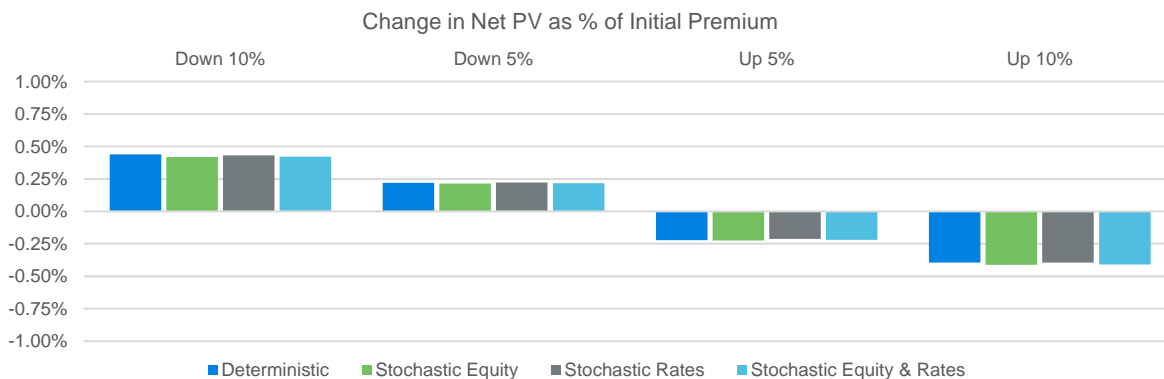
- Dynamic lapse behavior, through the discounting of the policyholders' expected benefits in the moneyiness formula
- Option budgets that are assumed to vary with projected interest rates through the estimated book yield formula used throughout this illustration

Figure 4 shows that, despite the sensitivity of the liability value to shocks in the projected interest rates, the change from the base case for each of the approaches is almost identical.

Equity sensitivity

Figure 5 shows the change in unitized net present values, compared to the baseline results, for several shocks to equity prices.

FIGURE 5: EQUITY SENSITIVITY



There is sensitivity to the equity index level through the value of the open index segment. A reduction in the expected value of the next credit increases the expected claims and vice versa, although due to discounting the exchange is not dollar for dollar. There is a second-order effect through the dynamic lapse function, due to change in the surrender value.

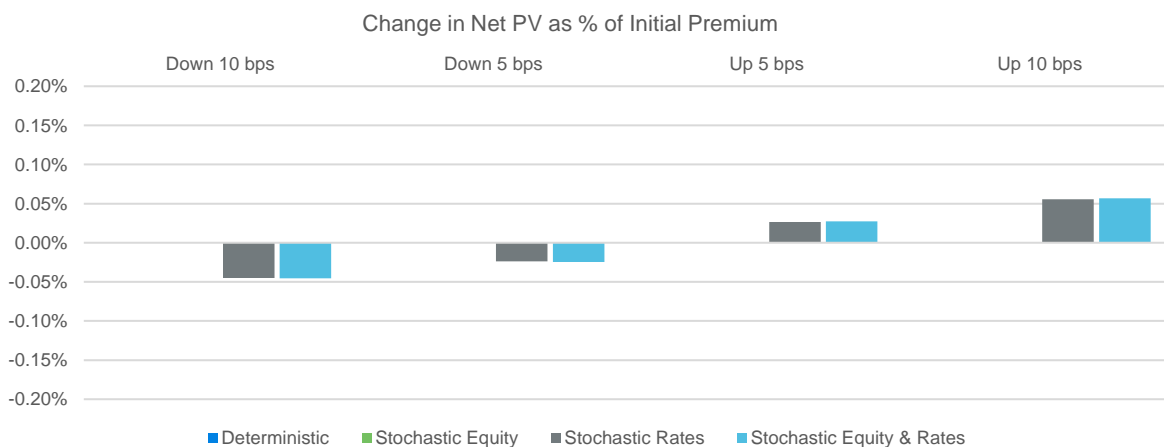
For the stochastic equity models, this sensitivity was simulated as an initial shock to the scenario returns. For the deterministic and stochastic interest rate-only models, where crediting is based on the option budget, the equity sensitivities were simulated by adjusting the first credit by the change in the Black-Scholes price given the equity shock.

As with the interest rate sensitivities, the equity sensitivities are nearly identical across each of the approaches.

Interest rate volatility sensitivity

Figure 6 shows the change in unitized net present values, compared to the baseline results, for several shocks to the Hull-White interest volatility parameter.

FIGURE 6: INTEREST RATE VOLATILITY SENSITIVITY



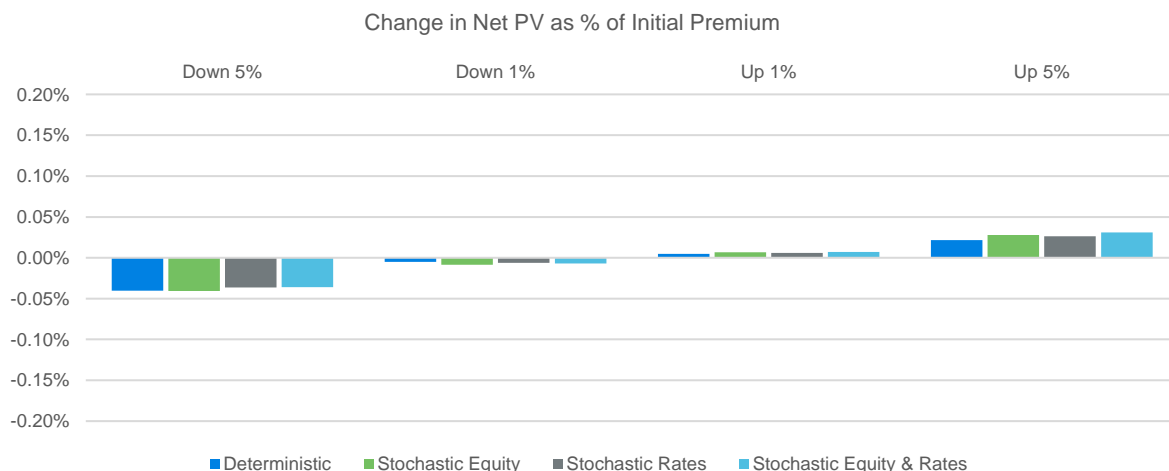
For FIA with GLWBs in general, there may be sensitivity to interest rate volatility through dynamic lapse policyholder behavior, and due to guaranteed minimum cap rates, which may come into play in low interest rate scenarios where the book yield is too low to support the target spread. The magnitude of the dynamic lapse effect is dependent on the responsiveness of the dynamic lapse function. Companies have different views on this, and some may not assume any dynamic lapse effect at all. The magnitude of the minimum cap rate effect is dependent on the product guarantees. We note that there would be an offset to this effect in the base contract reserves, assuming consistent assumptions and methodology. That is, any decrease in expected GMxB claims when the minimum cap rate is in effect would be partially offset by higher expected account value releases. We have illustrated a product with a guaranteed minimum cap of 1%, with an equivalent option budget of 0.5%, so the guaranteed minimum cap does not factor in the calculation except in most extreme low interest rate scenarios. Thus, the rate volatility sensitivity shown here is mostly due to the dynamic lapse effect.

For the illustrated assumption and product features, the impact of changes in interest rate volatility is relatively small: the maximum change in the net present value for the given shocks is less than 0.06% of the initial premium, or about 1% of the baseline net present value.

Equity volatility sensitivity

Figure 7 shows the change in unitized net present values, compared to the baseline results, for several shocks to the level of equity volatility.

FIGURE 7: EQUITY VOLATILITY SENSITIVITY



This sensitivity represents a shock to both the scenario volatility and the Black-Scholes implied volatility used to determine the reset cap rates, which are assumed to be equal throughout this illustration. Due to the ability to reprice forward starting caps with the updated implied volatility, changes in the scenario volatility do not have an impact on the expected (mean) payoff of forward starting index segments. That is, the cap repricing algorithm is solving for a cap rate such that the discounted expected payoff equals the option budget, and changes to implied volatility affect the cap rates but not the expected payoff.

The small amount of sensitivity shown above is primarily due to the impact on the value of the open index segment, which the insurer cannot reprice. We note that, for cap designs, volatility skew is a bigger driver of value than the level of volatility. Open segments of participation rate strategies, for example, would be more sensitive to the volatility level. While we have not illustrated other strategy designs in this paper, we assert that the deterministic approach would be fully capable of capturing that effect through revaluation of the initial option value using Black-Scholes (or another model more suitable for valuing exotic strategies).

As with the equity and interest rate sensitivities, the sensitivity to the level of equity volatility is similar across market models.

CONCLUSION

For the illustrated product and assumptions, the analysis presented above has shown:

- All four approaches produce nearly identical baseline results.
- All four approaches produce nearly identical sensitivities to interest rates, equity prices, and the level of equity volatility.
- The interest rate sensitivity is significant, the equity price sensitivity is moderate, and the equity volatility sensitivity is small.
- While there are differences in the sensitivity to rate volatility (nonzero for the models with stochastic rates, zero for the models without stochastic rates), the sensitivity to rate volatility is relatively small.

While there is clearly market sensitivity, particularly to interest rate and equity levels, these results suggest that there is little to no optionality with respect to interest rates or equity levels for the illustrated product. Roughly speaking, the MRB value behaves more like a deep-in-the-money option than an at-the-money option.

We believe the results of this analysis would provide sufficient justification for using a deterministic approach to value the MRB associated with this product. While we believe these results will be robust to small differences in product features or assumptions compared to those we have modeled, it is important to note that these results may not hold in general. Each company should perform its own analysis on its products and assumptions before drawing any conclusions on the ability of any methodology to capture the key risks.

Appendix: Detailed Assumptions and Product Features

PRODUCT FEATURES

Index features

We have assumed that 100% of the contract funds are invested in a one-year, point-to-point cap strategy with annual reset of the cap rate. The cap rate is guaranteed to never fall below 1.00%. Given the other assumptions and baseline market parameters assumed below, the initial cap rate is 4.17%.

GLWB design

The illustrated GLWB has a benefit base with 7% annual compound roll-up until activation of the GLWB benefit.

The maximum withdrawal amounts are a percentage of the benefit base, and vary with attained age at activation. The maximum withdrawal rates are summarized in the table below.

Age at First WD	Max Withdrawal Rate (% of Benefit Base)
60	4.50%
65	5.00%
70	5.50%
75	6.00%
80	6.50%

Rider charges are 1.00% of the benefit base per annum.

Surrender charge schedule

Policy Year	1	2	3	4	5	6	7	8	9+
Surrender Charge	8%	7%	6%	5%	4%	3%	2%	1%	0%

MODEL OFFICE

The model office consists of males and females with issue ages 55, 65, and 75, with initial premiums distributed as shown in the table below.

Issue Age	Female	Male
55	20%	20%
65	20%	20%
75	10%	10%

ASSUMPTIONS

Option budgets

Option budgets are forecast using a book-yield-less-spread approach. Expected book yields are estimated assuming a fixed percentage of the portfolio is reinvested at new money rates, where new money rates are assumed to include net spreads in excess of risk-free rates. The formulae for this approach, and the parameters used in our analysis, are as follows, where t is the projection month:

$$NetBookYield_t = NetBookYield_{t-1} \cdot (1 - \lambda) + NewMoneyRate_t \cdot \lambda$$

$$NewMoneyRate_t = RiskFreeRate_t + AssetSpread$$

$$OptBudget_t = NetBookYield_t - PricingSpread$$

$$\lambda = \frac{1}{TurnoverPeriod}$$

$$NetBookYield_0 = 4\%$$

$$TurnoverPeriod = 120 \text{ months}$$

$$AssetSpread = 1\%$$

$$PricingSpread = 2\%$$

Under the deterministic interest rate models, the resulting option budgets are 2% in all periods. In the stochastic interest rate models the initial option budget is 2% and future option budgets vary by scenario.

GLWB activation and utilization

No partial withdrawals are assumed prior to the activation of the GLWB benefit. Upon activation of the GLWB benefit, the policyholder is assumed to take 100% of the maximum withdrawal amount.

The table below summarizes the assumed GLWB activation distribution for each issue age.

Issue Age	First GLWB Withdrawal		
	At Issue	5-year delay	10-year delay
55	0%	25%	75%
65	33%	33%	33%
75	75%	25%	0%

Surrender rates

Base surrender rates are assumed to be 1% per annum in the surrender charge period, 15% in the first year after the end of the surrender charge period, and 5% in each year thereafter, as summarized below.

Policy Year	1	2	3	4	5	6	7	8	9	10+
Base Lapse Rate	1%	1%	1%	1%	1%	1%	1%	1%	15%	5%

The base surrender rates are adjusted by dynamic lapse factors, calculated according to the formula below:

$$w_t = \max(0, \min(1, w_{base,t} \cdot DynFactor_t))$$

$$DynFactor_t = \left(\frac{SV_t}{PVB_t} \right)^{DynExp}$$

$$SV_t = \text{Surrender Value at time } t$$

$$PVB_t = \text{Present value of future GLWB withdrawals at time } t \text{ discounted at interest rate at time } t$$

$$DynExp = 0.75$$

This formula result in surrender rates with the following properties:

- When the present value (PV) of benefits equals the surrender value, the dynamic factor equals one, and the adjusted surrender rates equal the base surrender rates.
- As the ratio of the surrender value to present value of GLWB benefits approaches zero, the adjusted surrender rates also approach zero.
- The PV of GLWB benefits increases as interest rates decrease, so all else equal the surrender rates decrease as interest rates decrease, and vice versa.
- The surrender value increases when the index account increases, so all else equal positive equity performance will increase surrender rates and vice versa.

Mortality rates

Base mortality rates are assumed to follow the 2012 IAM Period ANB table.

Mortality improvement is projected indefinitely using scale G2 with a base year of 2012.

MARKET MODELS

Deterministic interest rates

$r = 3.00\%$ // i.e., constant interest rates

One-factor Hull-White interest rates

Dynamics

$$dr_t = (\theta_t - a r_t) dt + \sigma_r dZ$$

Illustrated Parameters

$$r_0 = 3.00\%$$

$$\sigma_r = 0.50\%$$

$$a = 5.0\%$$

$$\theta_t = \frac{df_t}{dt} + af_t + \frac{\sigma_r^2}{2a} (1 - e^{-2at})$$

$f_t = 3.00\%$ // i.e., expected forward rate is constant

Lognormal equity

Dynamics

$$\frac{dS_t}{S_t} = \left(r - \frac{\sigma_e^2}{2} \right) dt + \sigma_e dZ$$

Illustrated Parameters

$$\sigma_e = 15\%$$

$$r = 3.0\%$$

One-factor Hull-White plus lognormal equity*Dynamics*

$$\frac{dS_t}{S_t} = \left(r_t - \frac{\sigma_e^2}{2} \right) dt + \sigma_e dZ_1$$

$$dr_t = (\theta_t - a r_t) dt + \sigma_r dZ_2$$

$$dZ_1 dZ_2 = \rho dt$$

Illustrated Parameters

$$\sigma_e = 15\%$$

$$r_0 = 3.00\%$$

$$\sigma_r = 0.50\%$$

$$a = 5.0\%$$

$$f_t = 3.00\% // \text{i.e., expected forward rate is constant}$$

$$\theta_t = \frac{df_t}{dt} + af_t + \frac{\sigma_r^2}{2a} (1 - e^{-2at})$$

$$\rho = 0$$

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